4.3 Differentiation of Parametric Equations

Question Paper

Course	CIE A Level Maths
Section	4. Differentiation
Topic	4.3 Differentiation of Parametric Equations
Difficulty	Very Hard

Time allowed: 50

Score: /38

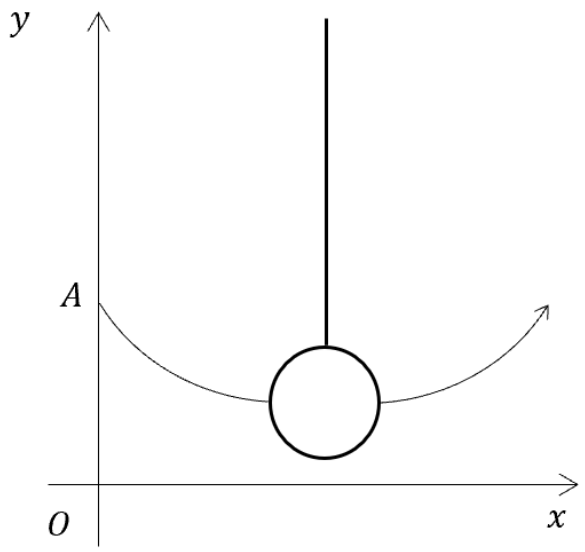
Percentage: /100

Question 1a

A crane swings a wrecking ball along a two-dimensional path defined by the parametric equations

$$x = 10t \qquad y = 4.9t^2 - 4.9t + 2 \qquad 0 \leq t \leq 1$$

as shown in the diagram below.



x and y are, respectively, the horizontal and vertical displacements in metres from the origin, O, and t is the time in seconds. Point A indicates the initial position of the wrecking ball.

(a) (i) Write down the height of the wrecking ball when it is at point A.

(ii) Find the shortest distance between the wrecking ball and the ground during its motion.

[4 marks]

Question 1b

(b) The destruction of a building requires the wrecking ball to strike it at a height of 1.4 m whilst on the upward part of its path.

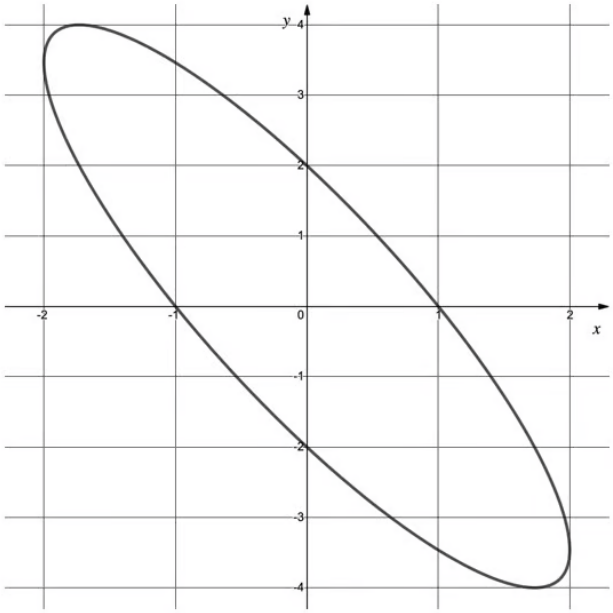
Find the horizontal distance from point A at which the ball hits the building.

[4 marks]

Question 2a

The graph of the ellipse E shown below is defined by the parametric equations

$$x = 2\cos\left(\theta + \frac{\pi}{3}\right) \qquad y = 4\sin\theta \qquad -\pi \leq \theta \leq \pi$$



(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

[3 marks]

Question 2b

(b) Find the equation of the tangent to E, at the point where $\theta = -\frac{\pi}{6}$, giving your answer in the form $y = a - bx$, where a and b are real numbers that should be given in exact form.

[4 marks]

Question 3

The curve C has parametric equations

$$x = 3t \qquad y = t + \frac{1}{t} \qquad t > 0$$

Find the equation of the normal to C at the point where C intersects the line $y = x$.

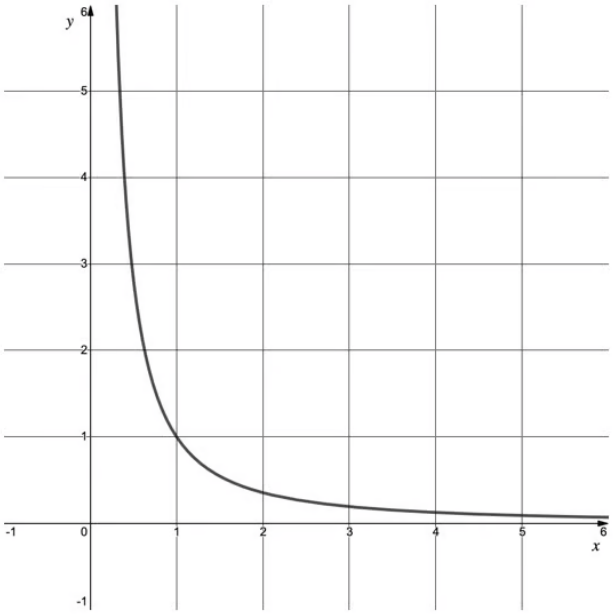
[9 marks]

Question 4

The graph of the curve defined by the parametric equations

$$x = e^{2t} \qquad\qquad y = e^{-3t}$$

is shown below.



(i) Verify that the graph passes through the point (1 , 1).

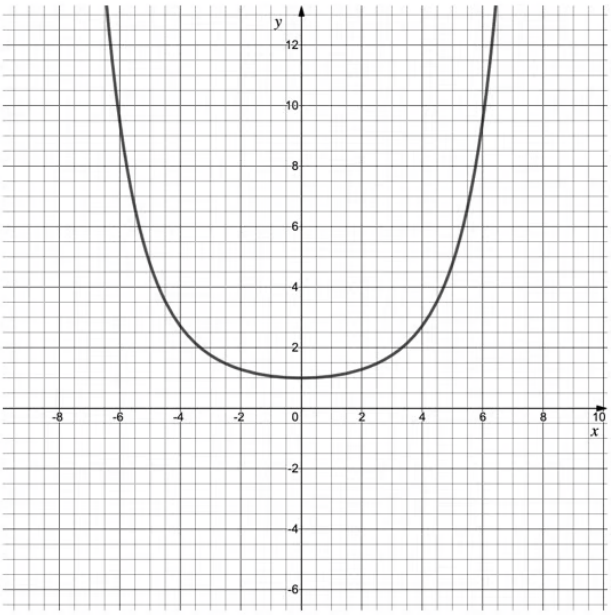
(ii) Prove that the line with equation $y = x$ is **not** the normal to the curve at the point (1 , 1).

[6 marks]

Question 5a

The diagram below shows a sketch of the curve defined by the parametric equations

$$x = 4t \qquad\qquad y = e^{t^2}$$



The tangents to the curve that pass through the origin meet the curve at points A and B

(a) Show that the values of t at points A and B are $t = -\frac{\sqrt{2}}{2}$ and $t = \frac{\sqrt{2}}{2}$.

[5 marks]

Question 5b

(b) Hence, or otherwise, show that the area of the triangle OAB is $2\sqrt{2}\, e^{\frac{1}{2}}$ square units.

[3 marks]